Design of a Dynamic Positioning System for a Moored Floating Platform using QFT Robust Control

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Abstract—**This paper describes the design of a dynamic positioning system for a moored floating platform by using robust control techniques, particularly Quantitative Feedback Theory (QFT). The goal is to minimize the drift resulting from the wave action by appropriate thrusters control. The model of the platform is a SIMO (single-input multiple-output) system, therefore an interesting question is that the plant has less degree of freedom for actuation and is difficult to control. The control problem of the underactuated system is solved by an iterative multi-stage sequential procedure. Simulation results are presented to demonstrate that the control achieves efficiently the dynamic positioning system. Therefore robust techniques based on QFT methodology constitute attractive alternatives in the application of positioning control of an underactuated marine system.**

Keywords-dynamic positioning system; underactuated system; robust control; multivariable control; floating platform

I. Introduction

The marine control problem described in this paper is commonly known as dynamic positioning system, which deals with control systems for station-keeping and low speed maneuvering, which simultaneous control the three horizontal motions (surge, sway, and yaw). It is defined a dynamically positioning vessel ([1]) as a free floating vessel which maintains its position, fixed location or predetermined track, exclusively by means of thrusters. In cases where anchors are used, these systems are referred to as position mooring system. For a free floating vessel in dynamic positioning the thrusters are the prime actuators for station-keeping, while for a position mooring system the assistance of thrusters are only complementary since most of the position keeping is provided by a deployed anchor system.

In particular, the offshore system studied in this paper consists of a moored floating platform. Position mooring and dynamic positioning are required in many offshore oil and gas field operations, such as drilling, pipe-laying, tanking between ships, and diving support (more applications can be seen in [2]). Therefore, these platforms require a high level of precision in the positioning for optimal operations. In addition, they are subject to environmental charges combined of waves, wind and currents, which affect such the stability as the positioning. Therefore robustness to plant uncertainties as well as rejection to environmental disturbances are important features of a dynamic positioning system. In addition, the system has lees degree of freedom for actuation, thus a control problem of an underactuated system is raised.

The control problem of dynamic positioning of underactuated marine systems has been examined and analyzed in the specialized literature, in which is possible to find several robust control methods. The LQG design technique was first applied to dynamic positioning by ([3], [4]) and ([5], [6]). Later Grimble and co-authors suggested both H_∞ and μ-methods for filtering and control ([7], [8]). After 1995, nonlinear PID control, passive observer design and observer backstepping design have been applied with good results ([9], [10], [2]). Other techniques are analyzed for example, in [11], an overview of a linear matrix inequality (LMI) approach to the multiobjective synthesis of linear output-feedback controllers is presented. In [12] the problem was formulated in the framework of a multimodel-based design of the H_∞ control law with pole region constraints. In [13], this system is used to validate the results obtained in the study about synthesis of reduced-order controllers based on LMI optimization.

Quantitative Feedback Theory (QFT) is a frequency domain robust-design methodology for control systems where the plant is uncertain and/or there are disturbances acting on the plant. Thus, the feedback control of the platform is a good example for using the QFT technique. The QFT technique was developed by Horowitz ([14]), and since the beginning has attracted considerable interest in both theory ([15]) and engineering applications ([16]), such as aeronautic, aerospace industry ([17]), robotic ([18], [19], [20]), electronic and electrical engineering ([21]), etc. In [22] Horowitz published a global vision of the method for the solution of scalar, multivariable, linear, non-linear and invariant/time varying uncertain systems is given. Initially QFT has not been very common in marine systems. Our group has applied this technique in the fast ferries stabilization problem ([23], [24]) with successful experimental results. Also it has been applied to develop a non-linear tracking control of an underactuated hovercraft ([25]).

Thus, the interest of the present work is focused on two questions mainly. First, the fact that robust control techniques based on QFT design are applied successfully to a typical marine control problem. On the other hand, from the point of view of theory of control, this paper shows that QFT is a feasibly methodology to solve the problem of rejection to disturbances in an underactuated system of a dynamic positioning problem.

The paper is organized as follows. Section 2 presents the model of the moored floating platform. Section 3 formulates the control problem. Section 4 describes the principles of QFT methodology. Section 5 develops the control technique employed and Section 6 gives the simulation results. Finally, Section 7 ends up with the conclusions.

II. MODEL OF THE PLATFORM

The system consists of a floating platform that is anchored to the bottom of the ocean and equipped with two thrusters (Fig. 1). The model employed is a model-scale platform in 1:100 scale described in [26]. The objective is to achieve an appropriate thrusters control in order to minimize the drift and angular deviation resulting from the wave action.

Figure 1. Moored floating platform.

The model of the system has two outputs y (the horizontal drift Y and angular deviation from the vertical axis ϕ), one control input u (the force delivered by the thrusters F_u) and two disturbance inputs d (the force F and the torque M from the wave action). Therefore a control structure of a single degree of freedom (DOF) for a SIMO (single-input, multiple-output) system is presented, with one single input F_u and two outputs (Y,ϕ).The platform dynamics are described by state-space equations:

$$\dot{x} = Ax + B\begin{pmatrix} F \\ M \\ F_u \end{pmatrix};\begin{pmatrix} Y \\ \phi \end{pmatrix} = Cx \quad (1)$$

Where x is the state vector, A, B and C are the following matrixes:

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.1010 & -0.1681 & -0.04564 & -0.01075 \\ 0.06082 & -2.1407 & -0.05578 & -0.1273 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.1179 & 0.1441 & 0.1476 \\ 0.1441 & 1.7057 & -0.7557 \end{pmatrix}; C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad (2)$$

In addition, the dynamic of the thrusters are modelled by the first-order transfer function $G_{act}(s)$:

$$F_u = G_{act}(s)\cdot u \qquad G_{act}(s) = \frac{1}{0.7s+1}\,. \quad (3)$$

where u is the control input and F_u the actual force delivered by the thrusters.

Regarding disturbances, the action from the wave is considered as a force F and a torque M. The force F consists of two components $F=F_1+F_2$. The spectral energy of F_1 is beyond 5 rad/s, and F_2 is concentrated between 0 and 1 rad/s.

The platform system can be expressed in transfer function model, by the following:

$$\begin{pmatrix} Y(s) \\ \phi(s) \end{pmatrix} = \begin{pmatrix} p_{11}(s) & p_{12}(s) & p_{13}(s) \\ p_{21}(s) & p_{22}(s) & p_{23}(s) \end{pmatrix}\cdot\begin{pmatrix} F(s) \\ M(s) \\ F_u(s) \end{pmatrix} \quad (4)$$

Where p_{ij}(s) defines a transfer function, in which for each case j=F, M, F_u is the input, and i= Y, ϕ is the output:

$$p_{11}(s) = \frac{Y(s)}{F(s)} = \frac{0.1179s^2 + 0.0135s + 0.2282}{s^4 + 0.1729s^3 + 2.25s^2 + 0.1018s + 0.2264} \quad (5)$$

$$p_{12}(s) = \frac{Y(s)}{M(s)} = \frac{0.1441s^2 + 7.655\cdot 10^{-6}s + 0.0217}{s^4 + 0.1729s^3 + 2.25s^2 + 0.1018s + 0.2264} \quad (6)$$

$$p_{13}(s) = \frac{Y(s)}{F_u(s)} = \frac{0.1476s^2 + 0.0269s + 0.443}{s^4 + 0.1729s^3 + 2.25s^2 + 0.1018s + 0.2264} \quad (7)$$

$$p_{21}(s) = \frac{\phi(s)}{F(s)} = \frac{0.1441s^2 - 5.502\cdot 10^{-6}s + 0.0217}{s^4 + 0.1729s^3 + 2.25s^2 + 0.1018s + 0.2264} \quad (8)$$

$$p_{22}(s) = \frac{\phi(s)}{M(s)} = \frac{1.706s^2 + 0.0697s + 0.1810}{s^4 + 0.1729s^3 + 2.25s^2 + 0.1018s + 0.2264} \quad (9)$$

$$p_{23}(s) = \frac{\phi(s)}{F_u(s)} = \frac{-0.7557s^2 - 0.0427s - 0.0673}{s^4 + 0.1729s^3 + 2.25s^2 + 0.1018s + 0.2264} \quad (10)$$

It can be proved that the horizontal drift Y suffers a more notable influence by the input disturbance F, while the input disturbance M causes a greater incidence to the angular deviation ϕ. Finally, a study of controllability and observabilty ([31]) confirms that the system is state controllable and observable, and completely input-output controllable.

III. CONTROL PROBLEM FORMULATION

A control system must perform mainly three functions in a marine vehicle. The first is to assure stability, the second is to attenuate seaway-induced motions, and the third is to assure the safety of the vehicle. For the particular moored floating platform system, the following control objectives ([26]) for the model-scale platform in study are required:

- -Reduce the drifting action F2 by the actuators control.
- -Maintain the horizontal drift |Y| < 0.025m.
- -Maintain the angular deviation |ϕ| < 0.07 degrees.
- -Keep |Fu| < 0.25 N.

- -Make sure that the thrusters have no response to the high-frequency component F1.

For design purposes, the system transfer function (4) can be described as:

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{P_{plant}}(s)\cdot u + \boldsymbol{P_d}(s)\cdot \boldsymbol{d} \\ u &= -\boldsymbol{G_{control}}(s)\cdot \boldsymbol{y} \end{aligned} \tag{11}$$

where $\boldsymbol{y} = [Y, \phi]^T$ is the output plant, $\boldsymbol{P_{plant}}(s)$ is a transfer functions matrix (2x1) that connects the input u with the output $\boldsymbol{y}$, and $\boldsymbol{P_d}(s)$ is a transfer functions matrix (2x2) that connects the disturbance d with the output y. In explicit form, it yields:

$$\begin{aligned} \begin{pmatrix} Y \\ \phi \end{pmatrix} &= \begin{pmatrix} p_{13}(s)\cdot G_{act}(s) \\ p_{23}(s)\cdot G_{act}(s) \end{pmatrix} \cdot u + \begin{pmatrix} p_{11}(s) & p_{12}(s) \\ p_{21}(s) & p_{22}(s) \end{pmatrix} \begin{pmatrix} F \\ M \end{pmatrix} \\ u &= -\boldsymbol{G_{control}} \cdot \begin{pmatrix} Y \\ \phi \end{pmatrix} \end{aligned} \tag{12}$$

Figure 2 displays schematically the feedback system described by (12). Thus, the control problem stated consist of a dynamic positioning system of a system with 1 degree of freedom (DOF), one input and two outputs (SIMO), and disturbances at plant's output.

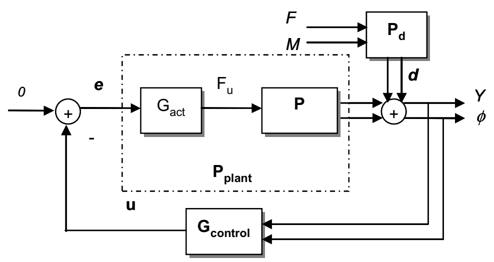

Figure 2. Single DOF SIMO system with disturbances at the plant's output.

Once the equations of the system are formulated, the control problem can be formally stated as following:

Given the system (11), where $\boldsymbol{P_{plant}}$ is a LTI 2x1 belonging to a set $\{P_{plant}\}$, $\boldsymbol{d}$ is a perturbation that belongs to a given set $\{d\}$, $e(\omega)$ a specification vector, and ω_h the frequency such that the specification vector is applicable for $\omega \le \omega_h$, design a controller $G_{control}$ such that simultaneously: *a)* the system $\boldsymbol{P_{plant}}$ is stable; *b)* decreases the plant output due to disturbances, i.e, for all d, the plant output $\boldsymbol{y}=[Y,\phi]^T$ is bounded by

$$\begin{aligned} |Y(t)| &\le e_1(t) = 0.025 \text{ m} \\ |\phi(t)| &\le e_2(t) = 0.07 \text{ degrees} \end{aligned} \tag{13}$$

An interesting question is added to the position control design because the plant has less degree of freedom for actuation, that is, it is an underactuated system, and is more difficult to control.

Taking into account all aforementioned, the challenge is to study the effectiveness of the QFT technique to accomplish the dynamic positioning of an underactuated system.

IV. QFT METHODOLOGY

QFT is a frequency domain design methodology that was introduced by Horowitz ([14]). The foundation of QFT is the fact that feedback is primarily needed when the plant is uncertain and/or there are disturbances acting on the plant. Thus, the feedback control of the floating platform is a good example for using the QFT technique because the system presents disturbances at plant's output, corresponding to the force and moment induced by the seaway.

The QFT design procedure involves four basic steps: generation of plant templates, computation of QFT bounds, design of the controller (loop shaping), and analysis of the design.

In this particular case, it is not necessary to generate plant templates, since the platform model is not considered a set of parametric uncertain system but a defined system. Thus, as a first step, QFT converts close-loop magnitude specifications into magnitude constraints on a nominal open-loop function (called QFT bounds). A nominal open-loop function $L(j\omega)$ is then designed to satisfy simultaneously the plant constraints as well as to achieve nominal closed-loop stability (loop shaping). The open loop function $L(j\omega)$ is the product of the controller transfer function $G(j\omega)$ and the plant transfer function $P(j\omega)$.

In any QFT design, it is necessary to select a frequency array for computing bounds. In the case of the platform system, the range of frequencies belongs to the seaway spectrum $\omega_0 \in [0.1, 10]$. To begin with, the formulation of what is the required behavior of the closed-loop system is necessary. These specifications must be given in terms of frequency response. QFT closed-loop specifications used are the gain and phase margin stability (14), the output disturbance rejection or sensitivity reduction (15), and the control effort (16):

$$\left| \frac{P(j\omega)\cdot G(j\omega)}{1 + P(j\omega)\cdot G(j\omega)} \right| \le \lambda(\omega); \forall \omega > 0, \tag{14}$$

$$\left| \frac{1}{1 + P(j\omega)\cdot G(j\omega)} \right| \le \delta_s(\omega); \forall \omega \le \omega_c, \tag{15}$$

$$\left| \frac{G(j\omega)}{1 + P(j\omega)\cdot G(j\omega)} \right| \le \delta_c(\omega); \forall \omega \le \omega_c, \tag{16}$$

The robust stability specification is related to the relative stability margins (phase margin PM and gain margin GM), through the following expressions:

$$\begin{aligned} GM &= 1 + \frac{1}{\lambda} \\ PM &= 180 - \frac{180}{\pi}\cdot \cos^{-1}\left(\frac{0.5}{\lambda^2} - 1 \right) \end{aligned} \tag{17}$$

For the particular case of the design of the dynamic positioning system for the moored platform model, the specifications (13) $e=[e_1(t), e_2(t)]^T$ are given in temporal domain. Therefore, it is necessary to translate temporal constraints into frequency domain specifications. Translating time-domain criteria into frequency domain specifications is not a trivial problem. There is no one-to-one translation, but from a practical point of view, good translations do exist. In this case, the model based technique ([28]) is employed. This method consists of assuming a plant and a controller model structure, and for a given input (generally a step), searching for the parameters of the assumed plant and controller models, and using the maximum or minimum of the amplitude of the resulting transfer function frequency response. Following this idea, from the frequency response, the estimation of δ_{sY} and $\delta_{s\phi}$ are the following:

$$\delta_{sY} = \max\left|Y(j\omega)\right| = 3 \tag{18}$$

$$\delta_{s\phi} = \max\left|\phi(j\omega)\right| = 15 \tag{19}$$

For the other specifications, the procedure is the same.

V. DESIGN OF A CONTROLLER FOR A SIMO SYSTEM

The platform model is an underactuated system. Previous approaches have tried to solve the problem by approximating the *n*x*n* MIMO solution to the problem *n*x*m* with more outputs than control inputs. This section solves the *1*x*2* SIMO system that states the dynamic positioning problem.

The control law of the system in Fig. 2 is

$$\boldsymbol{G_{control}}(s) = \left(k_1(s) \quad k_2(s)\right) \tag{20}$$

Solving the system given by equations (12) and (20), and using the notation $P_{plant}^{-1} = \left(\hat{p}_{13} \quad \hat{p}_{23}\right)$, it yields:

$$\begin{aligned}&\left(\hat{p}_{13} + k_1\right)Y + \left(\hat{p}_{23} + k_2\right)\phi = \left(\hat{p}_{13}p_{11} + \hat{p}_{23}p_{21}\right)F + \\ &+ \left(\hat{p}_{13}p_{12} + \hat{p}_{23}p_{22}\right)M\end{aligned} \tag{21}$$

The design process is based on this equation which depicts the SIMO system of Figure 2 with one input and two outputs. The equation presents two unknown quantities (the controllers k_1 and k_2). In this way, the problem of the controllers design is solved by a multi-stage procedure by transforming the problem into the design of two sequential SISO systems. The idea is schematically depicted in Figure 3.

For each stage and consequently, for each SISO system, once stability and performance bounds have been computed, the next step in a QFT procedure involves the design (loop shaping) of a nominal function that meets its bounds. The nominal loop $L(j\omega)$ has to satisfy the worst case of all bounds. The MATLAB toolbox ([29]) includes an interactive design environment. Once the controller parameters are designed by using QFT design, the system in closed loop dynamic (Fig. 2) is simulated in order to prove if the control meets the specifications.

VI. CONTROL DESIGN RESULTS

According to the methodology explained, finally the control design procedure was completed in five stages. In this work it is shown the results of the QFT design of the two last stages in which the definitive controller (k_1, k_2) is determined. Specifically, the fourth stage corresponds to the k_2 design, and the fifth stage to k_1 design. To finish, simulations of the system in closed loop are carried out in order to examine if the positioning system is achieved. Thus, temporal responses are shown.

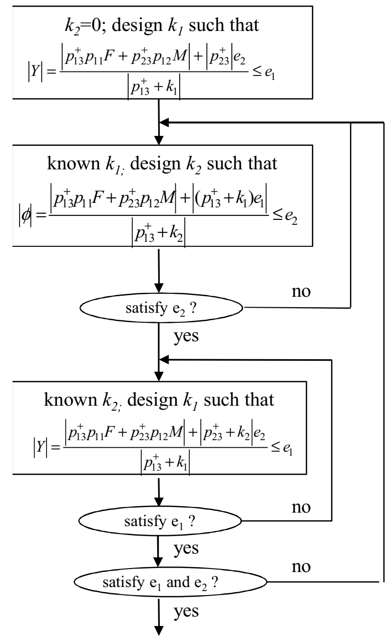

Figure 3. Scheme of the SIMO QFT design.

A. Design of the Controller k_2. Fourth Stage

The specifications for robust stability and performance bounds fixed for the second SISO QFT design guarantee adequate gain margins and output disturbance rejection:

- Gain and phase margins λ_2= 2.8 (22)
- Sensitivity reduction δ_{s2}=1 (23)
- Control effort δ_{c2}=16 (24)

The stability margins implies a robust stability, according to (17), with at least 1+(1/ λ_2)=2.61 lower gain margin, and 180º-cos^{-1}(0.5/ λ_{22}-1)=20.57º lower phase margin. This makes the following inequality satisfies for all frequencies:

$$\left|\frac{\hat{p}_{23}\cdot k_2}{1 + \hat{p}_{23}\cdot k_2}\right| \le \lambda_2 = 2.8 \qquad \omega \ge 0 \tag{25}$$

The bounds at low frequencies ($\omega \le$ 10 rad/s) are calculated in order to satisfy the inequality for disturbance rejections and control effort respectively:

$$\left|\frac{1}{1 + \hat{p}_{23}\cdot k_2}\right| \le \delta_{s2} = 16 \quad \omega \le 10\, rad/s \tag{26}$$

$$\left|\frac{k_2}{1 + \hat{p}_{23}\cdot k_2}\right| \le \delta_{c2} = 15 \quad \omega \le 10\, rad/s \tag{27}$$

The control k_2 must be designed such that open loop function $L_2(j\omega)$ given in (28) satisfies the worst case of all bounds (intersection). The controller designed (29) is a strictly proper third order filter, therefore it avoids high frequency effects.

$$L_2(j\omega) = k_2(j\omega)\cdot 1/\hat{p}_{23}(j\omega) \tag{28}$$

$$k_2(s) = \frac{(0.74s^2 + 0.008s + 0.05)}{(0.001s^3 + 0.18s^2 + 0.59s + 1)} \tag{29}$$

Figure 4 shows the Nichols chart of the open-loop function $L_2(j\omega)$ with the given specifications. It is shown that this controller accomplishes the specifications.

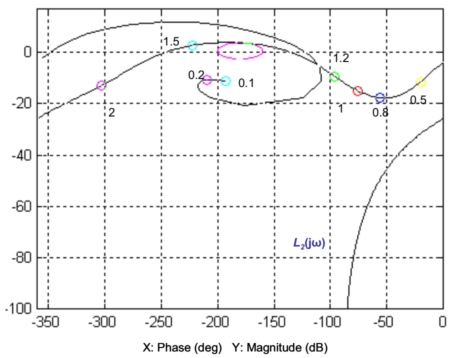

Figure. 4. Nominal open loop function $L_2(j\omega)$ and intersection of all bounds, for second SISO system.

B. Design of the Controller k_1. Fifth Stage

The close-loop specifications chosen for the first SISO QFT design are:

- Gain and phase margins λ_1= 2.8 (30)
- Sensitivity reduction δ_{s1}=3 (31)
- Control effort δ_{c1}=15 (32)

The value λ_1= 2.8 (MG=2.61 dB, MF=20.57º) for the stability margins is chosen arbitrarily and implies the following inequality is satisfied for all frequencies:

$$\left|\frac{\hat{p}_{13}\cdot k_1}{1+\hat{p}_{13}\cdot k_1}\right| \le \lambda_1 = 2.8 \qquad \omega \ge 0 \tag{33}$$

The values that compute the disturbance rejection bounds (δ_{s1}=3) and control effort (δ_{c1}=15) at low frequencies ($\omega \le 10$ rad/s) guarantee an adequate sensibility reduction and control effort respectively. Thus, the inequalities for those specifications are:

$$\left|\frac{1}{1+\hat{p}_{13}\cdot k_1}\right| \le \delta_{s1} = 3 \quad \omega \le 10\, rad/s \tag{34}$$

$$\left|\frac{k_1}{1+\hat{p}_{13}\cdot k_1}\right| \le \delta_{c1} = 15 \quad \omega \le 10\, rad/s \tag{35}$$

Figure 5 depicts the loop shaping in the Nichols chart, with the bounds and the nominal open-loop function (36), where it is seen that $L_1(j\omega)$ lies outside the margin bounds at the corresponding frequencies. Therefore it is shown that the controller designed (37) meets the specifications. In addition a pole at high frequency is added in order to filter noise in practise, so the controller is strictly proper.

$$L_1(j\omega) = k_1(j\omega)\cdot 1/\pi_{13}(j\omega) \tag{36}$$

$$k_1(s) = -\frac{(0.74s + 0.4)}{(0.023s^2 + 0.42s + 1)} \tag{37}$$

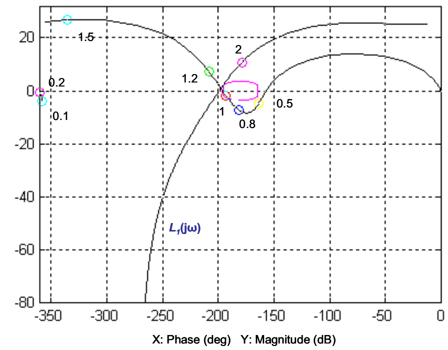

Figure. 5. Nominal open loop function $L_1(j\omega)$ and intersection of all bounds, for first SISO system.

C. Analysis of the Design.

Simulations of the SIMO control system is undertaken in order to evaluate the whole system performance and validate the controllers obtained. With this goal, temporal responses of the SIMO system (Figure 2) in closed loop dynamic are shown. Figures 6 and 7 compare respectively both outputs Y and ϕ in open-loop with the outputs in closed-loop considering the control law $\boldsymbol{u}=[k_1, k_2]^T$.

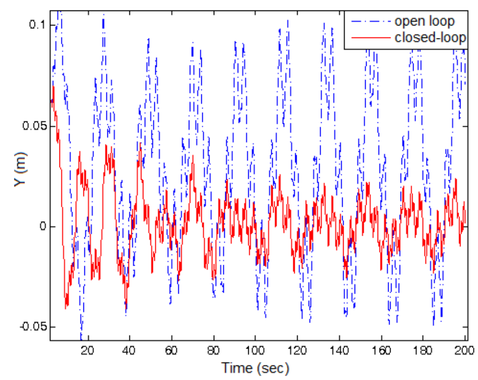

Figure 6. Comparison of temporal response $Y(t)$ in open loop (dashed line) and closed loop (solid line).

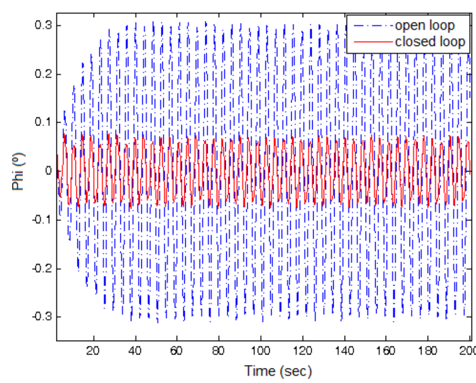

Figure 7. Comparison of temporal response $\phi(t)$ in open loop (dashed line) and closed loop (solid line).

As seen, the controller achieves the output $Y(t)$ gets into the range ±0.025m before t = 80 seconds. Regarding $\phi(t)$, it is observed that it remains the range given by the specification ± 0.007º from the beginning. Therefore, it is shown that the control meets the original specifications (13) and therefore, achieves the positioning system.

VII. CONCLUSIONS

This research has used a quantitative design technique (QFT) to design a dynamic positioning system for a moored floating platform. The system consists of two outputs (the horizontal drift and deviation angle), and one control input (the force by the thrusters). In addition it presents two disturbances inputs (forces and torques from wave action). Thus, the plant model is a single degree of freedom underactuated (SIMO) system with disturbances at plant input.

Several procedures have been analysed in order to synthesize the control for the SIMO plant. Finally, an approach based on robust QFT design is derived from the equation of the system. The idea consists of transforming the problem into two sequential SISO systems and breaking the design process into a series of iterative stages, in such a way that the solution in the first system is used in the design in the second system, and vice versa. Finally it is shown that QFT design is a robust method very suitable for the implementation, and that accomplishes the objectives efficiently. We have verified that this method is an attractive alternative to handle robust design of SIMO marine systems.

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